Normal states are determined by their facial distances

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Let M be a semi-finite W^*-algebra with normal state space $\mathfrak{S}(M)$. For any $\phi \in \mathfrak{S}(M)$, let

$$M_\phi := \{x \in M : x\phi = \phi x\}$$

be the centralizer of ϕ with center $\mathcal{Z}(M_\phi)$. We show that for $\phi, \psi \in \mathfrak{S}(M)$, the following are equivalent.

- $\phi = \psi$.
- $\mathcal{Z}(M_\psi) \subseteq \mathcal{Z}(M_\phi)$ and $\phi|_{\mathcal{Z}(M_\phi)} = \psi|_{\mathcal{Z}(M_\phi)}$.
- ϕ, ψ have the same distances to all the closed faces of $\mathfrak{S}(M)$.

We are then able to give an alternative proof of the following fact. Let G be a locally compact group. Let A be any one of the (complex) Banach algebras: $L_1(G)$, $M(G)$, $WAP(G)$, $LUC(G)$, $B(G)$, and $A(G)$, consisting of integrable functions, regular Borel complex measures, weakly almost periodic functions, bounded left uniformly continuous functions, positive definite functions, and positive definite functions vanishing at infinity, respectively, on G. We show that the metric semigroup

$$A_{+,1} := \{f \in A : f \geq 0 \text{ and } \|f\| = 1\}$$

(the convex structure is not considered) is a complete invariant for G.

This is a joint work with Anthony To-Ming Lau (Alberta) and Chi-Keung Ng (Nankai).

Keywords: W^*-algebras; normal states; F-algebras; facial structures; locally compact groups.

Solvability of a semilinear heat equation via a quasi scale invariance

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Classification theory on local in time solvability of nonlinear heat equations is investigated. Without assuming a concrete growth rate on a nonlinear term, we reveal the threshold integrability of initial data which classify existence and nonexistence of solutions via a quasi-scaling and its invariant integral. Global in time solvability is also studied. Typical nonlinear terms, for instance polynomial type, exponential type and its sum, product and composition, can be treated as applications. This is a joint work with Yohei Fujishima(Shizuoka University).

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Chaotic translations on weighted Orlicz spaces

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This is a joint work with K-Y. Chen, Prof. S. Öztop and Prof. S. M. Tabatabaie. Let G be a locally compact group, and let w be a weight on G. Let Φ be a Young function. We give some characterizations for translation operators to be topologically transitive and chaotic on the weighted Orlicz space $L^{\Phi}_{w}(G)$. In particular, transitivity is equivalent to the blow-up/collapse property in our case. Moreover, the dense set of periodic elements implies transitivity automatically.

Keywords: Chaos, Translation, Orlicz spaces.

On the boundedness of bilinear pseudo-differential operators of $S_{0,0}$-type

Naohito Tomita (Osaka University, Japan)

The Hörmander symbol class $BS^m_{0,0}$, $m \in \mathbb{R}$, consists of all $\sigma(x,\xi,\eta) \in C^\infty((\mathbb{R}^n)^3)$ such that

$$|\partial_x^\alpha \partial_\xi^\beta \partial_\eta^\gamma \sigma(x,\xi,\eta)| \le C_{\alpha,\beta,\gamma}(1+|\xi|+|\eta|)^m \tag{1}$$

for all multi-indices α, β, γ, and the bilinear pseudo-differential operators T_σ is defined by

$$T_\sigma(f,g)(x) = \frac{1}{(2\pi)^{2n}} \int_{(\mathbb{R}^n)^2} e^{ix\cdot(\xi+\eta)} \sigma(x,\xi,\eta) \widehat{f}(\xi)\widehat{g}(\eta)\, d\xi d\eta,$$

where f, g are Schwartz functions on $\mathbb{R}^n$ and $\widehat{f}, \widehat{g}$ are the Fourier transforms of f, g. In the first part of this talk, we consider the problem of determining the sharp order m (in (**??**)) to assure the $L^p \times L^q \to L^r$ boundedness. This part is based on a joint work with Akihiko Miyachi (Tokyo Woman's Christian University). In the second part, we concentrate on the $L^2 \times L^2 \to L^1$ boundedness, and look for weaker conditions than the classical one (**??**) to assure it. This part is based on a joint work with Tomoya Kato (Gunma University) and Akihiko Miyachi.

Two weight T1 theorem for fractional Riesz transforms

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In this talk we present our recent advances showing the two weight T1 theory for any fractional Riesz transforms when one of the measures is supported on a curve. An application of our theorem gives a characterization of the embedding problem for model spaces.

Zero product preservers and homomorphisms between matrix algebras

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In this talk, we give concrete description of the structures of ring, algebra and Jordan homomorphisms, and linear disjointness preservers between matrix algebras of different sizes. After giving full descriptions of ring, algebra and Jordan homomorphisms between matrices, we show that a linear map $\Phi : M_n \to M_r$ preserving zero products carries the form

$$\Phi(A) = S \begin{pmatrix} R \otimes A & 0 \\ 0 & \Phi_0(A) \end{pmatrix} S^{-1},$$

for some invertible matrices R in M_k, S in M_r and a zero product preserving linear map $\Phi_0 : M_n \to M_{r-nk}$ with range consisting of nilpotent matrices. When $\Phi(I_n)$ is diagonalizable, especially self-adjoint, normal, or an idempotent, we have $\Phi_0(X)\Phi_0(Y) = 0$ for all X, Y in M_n. If Φ preserves self-adjoint matrices, then we can assume $S^{-1} = S^*$, $R^* = R$ and $\Phi_0 = 0$. Similar results for double zero product preservers and orthogonality preservers are obtained..

Keywords: disjointness preserver, zero product, matrix spaces, matrix algebras.

Optimality and duality for complex multi-objective programming

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We consider a complex multi-objective programming problem (CMP). In order to establish the optimality conditions theorem of problem (CMP), we introduce the properties of optimal efficient solutions and scalarization techniques. Furthermore, the parametric dual and second-ordered parametric dual models are discussed, and their duality theorems are proved.

Keywords: multi-objective programming, efficient solutions, generalized convexity, duality problem

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The Birkhoff–von Neumann theorem version of row-stochastic matrices

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The Birkhoff–von Neumann (BNT) theorem states that every $n \times n$ doubly stochastic matrix, whose nonnegative real entries and each of rows and columns sums equal to 1, can be written as a convex combination of $n \times n$ permutation matrices. The BNT has been proved many times in numerous literature with a number of different methods, some inductive, some constructive, see [3, 4] for examples. In this work, we prove that every row-stochastic (RS) matrix of order $m \times n$ (the summation of entries of each row is equal to 1) can be written as a convex combination of n^m $\{0,1\}$-RS matrices by using a fundamental theorem: Hahn-Banach theorem. We also construct two algorithms for finding the coefficients of a convex combination of a row-stochastic matrix. This is a joint work with Huai-Xin Cao, Zhi-Hua Guo, Tsung-Lin Lee, and Ngai-Ching Wong.

Keywords: stochastic matrix, Birkhoff–von Neumann theorem, simplex method, heuristic method.

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